

Settling Conjectures 10.1, 10.2 and 10.3 on alternating plane graphs

Hanyu Yang – ORCID 0009-0005-0419-4070

Concept DOI 10.5281/zenodo.22269200 · Licence: MIT

Repository: <https://github.com/SeverinVisionary/alternating-plane-graphs>

Landing page: <https://severinvisionary.github.io/alternating-plane-graphs/>

*This PDF is a typeset rendering of **ARTIFACT.md**, the artifact overview, at commit **b489d8a7f3a5** on branch **main** of the repository above **with uncommitted local changes**. The text below is that file as it stands at that commit, and nothing in this PDF is authored here. The manuscript in **paper/** is a separate document and is not reproduced here.*

This repository is the evidence for three settled conjectures. It is meant to be deposited and read by someone with no prior contact with it, so this file says what is here, what is load-bearing, and how to verify it without trusting any claim made in prose.

For a one-page formatted overview see **ZENODO.md**.

Source problem: Althofer, Haugland, Scherer, Schneider and Van Cleemput, *Alternating plane graphs*, Ars Math. Contemp. **8** (2015) 337-363, DOI 10.26493/1855-3974.584.09a.

Results

Section 10 item	status	where
Conjecture 10.1 (no 2,Y-, no X,2-APG)	proved , unconditionally	CONJECTURE_10_1.md
asymptotic degree distribution	open	DENSITY.md
Conjecture 10.2 ((3,4,5)-APG for all $n \geq 20$)	settled ; every $n \geq 46$ closed here, 20..45 inherited from the paper	PUMPING_LEMMA_STATUS.md
Conjecture 10.3 (3-connected APG for all $n \geq 19$)	settled	CONJECTURE_10_3.md

Attribution – what is the source paper’s, what is re-verified here, and what is new – is collected in **ATTRIBUTION.md**.

This work was produced with substantial AI assistance, including the step that makes Conjecture 10.1 unconditional. The extent, and what was and was not taken on trust, is stated in **AI_DISCLOSURE.md**.

The (C2) qualification on 10.1 was removed on 2026-09-02. It had been the paper’s reading of Definition 2.1 at a bridge rather than a hypothesis written into it – Definition 2.1 lists four conditions and says nothing about bridges, but p. 339 asserts that an APG "is always at least 2-edge-connected, since plane graph with edge connectivity 1 contains a face that is adjacent to itself", which is the strict reading stated as a consequence in passing. **bridge_lemma.py** supplies the parity lemma and **conjecture_10_1.py** the count that closes it, so the theorem now holds under either reading, and does not depend on that sentence. This is a robustness result, not the closing of a gap the authors left open.

Verifying it

```
make deps      # pytest 8.4.2, and nothing else
make verify    # 1257 passed, 13 skipped, about 11 minutes
```

or, for the load-bearing gates only, about a minute:

```
make verify-fast
```

Recorded runtime for the figure above: Python 3.9.6 on macOS, 10m40s. Of the 13 skips, 11 need the published planar_code corpus, which is not redistributed here (see `NOTICE.md` and `confest.py`), one needs the optional `python-sat`, and one is an empty parameter set that follows from the corpus being absent. Nothing that is settled depends on any of them.

The core needs only the standard library. The proofs, the certificates and all three verifiers import nothing outside it; `pytest` is the runner. `numpy`, `sympy`, `z3` and `python-sat` appear only in optional search lanes and one archived analysis script, none of which any settled result rests on.

What is in the box

path	count	what it is
certificates/targets/	26	the Conjecture 10.2 witnesses, one per previously open order
certificates/ (all)	80 JSON	plus order-19, surgery, counterexample, seed and published witnesses, all as rotation systems
*.py modules	74	constructions, three independent verifiers, search lanes
test_*.py	77	the gates
results/	614	run artifacts, provenance only
figures/	12	SVG and TikZ for the five smallest certificates and the order-46 counterexample
*.md	32	the write-ups

Certificates are **rotation systems only** – no coordinates, no stored faces, no stored claim that a graph is an alternating plane graph. Degrees, faces, face sizes, bridges, connectivity and both alternation conditions are recomputed on every run by `verify.py`, `verify_darts.py` and `fast_apg_check.py`, which were written independently of each other. A corrupted certificate cannot pass by asserting its own correctness. The format is specified, with a dependency-free reference reader, in `FORMATS.md`; `test_formats.py` executes that reader out of the markdown against every certificate, so the specification cannot drift from the data.

Not load-bearing, and kept anyway

- `CLOUD_*_JOB.md` (9 files) are operator briefs for dispatching heavy searches to a Linux host. They are workflow instructions, not mathematics. They are kept because 27 cross-references

in the write-ups point at them and because they record what was attempted.

- **results/** holds run artifacts: search logs, calibration records and console captures. They are evidence about the *process*, never about the mathematics – every mathematical claim here is carried by a gate. Review transcripts are not part of this package; what independent review changed is recorded in `REVIEW.md`.
- **The search lanes** – `closed_map_search.py`, `plane_apg_search.py`, the `near_open_*` family – returned no hits. They are kept for their diagnoses, which are recorded in their own docstrings: the $(3,4,5)$ one-switch neighbourhood of every certificate is empty, and in the general class every local condition reaches zero while the whole residual penalty is genus.

Figures

`draw.py` computes a Tutte barycentric embedding from the rotation system, so a figure cannot depict a different graph from the one verified. `make figures` re-renders the committed SVG and TikZ in `figures/`, and `test_draw.py` requires the committed files to match a fresh render exactly.

The pipeline is honest about its own limit. These certificates are capped unrollings of a long periodic strip, so the barycentric solution crushes the interior exponentially in the order: minimum vertex separation runs $2.3\text{e-}03$ at order 46 down to $5.0\text{e-}12$ at order 110, and spurious crossings appear precisely where it falls through double precision. Above roughly order 50 the drawing is an artefact of the arithmetic rather than a picture of the graph, and `draw.py` refuses to write it.

Crossings say nothing about connectivity: the order-46 counterexample, which is *not* 3-connected, draws with none at all and at a wider separation than any 3-connected certificate of similar order. Figures for the large orders need a schematic of the cap-strip-cap decomposition instead, which is not built.

What is taken on trust

Stated in full in `CONJECTURE_10_3.md`, and short:

- that the five order-19 graphs are *all* the alternating plane graphs on 19 vertices. Irrelevant to the positive answer – one verified 3-connected witness settles the order, and all five are verified here. It would matter only for a refutation.
- that the published `planar_code` witnesses are the paper’s graphs. They are decoded and re-verified here, not taken on faith, and every result is independently reconstructible without them.

Before this is deposited

Nothing legal is outstanding. No third-party bytes are redistributed: the published `planar_code` corpus is not included, each of its graphs is re-expressed in this repository’s own format, and the digests of the originals are kept for verification. See `NOTICE.md`.

What remains is editorial. The manuscript in `paper/` now compiles – `paper/apg.pdf`, 11 pages, built with Tectonic – but it has had no peer review and its density section paraphrases the source problem rather than quoting it. See `paper/README.md`.